

# Bethe-Salpeter equation for diquarks in color-flavor locked phase of cold dense QCD

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The Bethe-Salpeter equations for diquarks with the quantum numbers of the Nambu-Goldstone bosons are analyzed in the color-flavor locked phase of cold dense QCD with three quark flavors. The decay constants and the velocities of the Nambu-Goldstone bosons are calculated in the Pagels-Stokar approximation. It is also shown that, in contrast with the case of dense QCD with two flavors, there are no massive radial excitations with quantum numbers of the Nambu-Goldstone bosons in the color-flavor locked phase. The role of the Meissner effect in the pairing dynamics of diquarks is explained.

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## I. INTRODUCTION

Quark matter at high density is a color superconductor. It has been the subject of many studies for the last few years. This recent increased activity was initiated by the observation [1,2] that the color superconducting order parameter could be much larger than previously thought (for old studies, see Refs. [3,4]). Since then, many new results have appeared [5–20].

Because of asymptotic freedom, QCD becomes a weakly interacting theory at high densities. This allows one to obtain some rigorous results for dense quark matter in the asymptotic limit. Of course, from the viewpoint of phenomenology, it is most desirable to have a theory valid at intermediate densities that could be produced in heavy ion collisions or could exist in nature (for example, inside neutron or quark stars). This dilemma is partially resolved by studying predictions of the theory at high densities and, then, extending their validity as far as possible to the region of interest [5–11]. Notice that all the heavy quark flavors could be safely omitted from the analysis when probing the quark matter at realistic intermediate densities. As a result, one arrives at a model of dense QCD with either two (“up” and “down”) or three (“up,” “down,” and “strange”) flavors.

In this paper, we deal with the problem of the diquarks with the quantum numbers of the Nambu-Goldstone (NG) bosons in cold dense QCD with three flavors. (A similar problem in the case of two flavors was considered in Refs. [21,22].) The ground state of dense quark matter with three flavors is a color superconductor in the so-called color-flavor

locked (CFL) phase [12]. It is remarkable that chiral symmetry in such a phase is broken and most of the quantum numbers of physical states coincide with those in the hadronic phase. It was tempting, therefore, to suggest that there might exist a continuity between the two phases [13].

The dynamical symmetry breaking is caused by the celebrated Cooper instability in the pairing dynamics of quasiparticles around the Fermi surface. As a result, the original gauge symmetry  $SU(3)_c$  and the global chiral symmetry  $SU(3)_L \times SU(3)_R$  break down to the global diagonal  $SU(3)_{c+L+R}$  subgroup [12]. Out of a total of 16 (would be) NG bosons, eight are removed from the physical spectrum by the Higgs mechanism, providing masses to eight gluons. The other eight NG bosons show up as an octet [under the unbroken  $SU(3)_{c+L+R}$ ] of physical particles. In addition, the global baryon number symmetry as well as the approximate  $U(1)_A$  symmetry also get broken. As a result, an extra NG boson and a pseudo NG boson appear in the low energy spectrum. These particles are both singlets under  $SU(3)_{c+L+R}$ .

The low energy effective theory in the CFL phase was formulated in Refs. [23–28]. Moreover, all parameters that define the low energy action of the (pseudo) NG bosons were calculated in the limit of the asymptotically large chemical potential. The elegant method of Refs. [24–28] is based on matching some vacuum properties (such as vacuum energy and gluon screening) of the effective and microscopic theories. While being rather efficient for some purposes, such a method is very limited when it comes to determining the spectrum of bound states other than NG bosons. The other approach used to study (diquark) bound states is based on the Bethe-Salpeter (BS) equation. In particular, the analysis of the BS equation seems to be the only way to test the conjecture of Ref. [14] about the existence of an infinite tower of

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massive diquark states in the color superconducting phase of dense quark matter. In two flavor QCD, such a conjecture proved to be partially true [21,22]. Here we generalize the method of Refs. [21,22] to the case of the CFL phase in QCD with three flavors.

This paper is organized as follows. In Sec. II, we describe the model and introduce the notation. Then, in Sec. III, we review the approach of the Schwinger-Dyson equation in the color superconducting phase of three flavor QCD. In Sec. IV, we derive the Ward identities for the quark-gluon vertex functions, corresponding to the broken generators of global and gauge symmetries. We outline the general derivation of the Bethe-Salpeter equation and present its detailed analysis for the diquark NG bosons in Sec. V. The decay constants and velocities of the NG bosons are calculated in Sec. VI. In Sec. VII, we discuss the problem of massive radial excitations in the diquark channel with quantum numbers of the NG bosons. Finally, our conclusions are given in Sec. VIII.

## II. MODEL AND NOTATION

As we mentioned in the Introduction, the original  $SU(3)_c \times SU(3)_L \times SU(3)_R$  symmetry of three flavor dense QCD breaks down to the global diagonal  $SU(3)_{c+L+R}$  subgroup. The condensate in the CFL phase is given by the vacuum expectation value of the following diquark (antidiquark) field [12]:

$$\langle 0 | (\bar{\Psi}_D)_i^a \gamma^5 (\Psi_D^C)_j^b | 0 \rangle = \kappa_1 \delta_i^a \delta_j^b + \kappa_2 \delta_i^a \delta_j^b, \quad (1)$$

where  $\Psi_D$  and  $\Psi_D^C = C \bar{\Psi}_D^T$  are the Dirac spinor and its charge conjugate spinor, and  $C$  is a unitary matrix that satisfies  $C^{-1} \gamma_\mu C = -\gamma_\mu^T$  and  $C = -C^T$ . The complex scalar functions  $\kappa_1$  and  $\kappa_2$  are determined by dynamics. Here we explicitly display the flavor ( $i, j = 1, 2, 3$ ) and color ( $a, b = 1, 2, 3$ ) indices of the spinor fields. Notice, however, that the CFL phase mixes color and flavor representations. As a result, the notions of ‘‘color’’ and ‘‘flavor’’ become essentially indistinguishable in the vacuum. In passing, we also note that the order parameter in Eq. (1) is even under parity.

By following Ref. [24], we introduce the color-flavor locked Weyl spinors [octets and singlets under  $SU(3)_{c+L}$  and  $SU(3)_{c+R}$ , respectively] to replace the spinors of a definite color and flavor:

$$\psi^A = \frac{1}{\sqrt{2}} \mathcal{P}_+ (\Psi_D)_a^i (\lambda^A)_i^a, \quad \psi = \frac{1}{\sqrt{3}} \mathcal{P}_+ (\Psi_D)_a^i \delta_i^a, \quad (2)$$

$$\tilde{\psi}^A = \frac{1}{\sqrt{2}} \mathcal{P}_- (\Psi_D^C)_j^b (\lambda^A)_b^j, \quad \tilde{\psi} = \frac{1}{\sqrt{3}} \mathcal{P}_- (\Psi_D^C)_j^b \delta_b^j, \quad (3)$$

$$\phi^A = \frac{1}{\sqrt{2}} \mathcal{P}_- (\Psi_D)_a^i (\lambda^A)_i^a, \quad \phi = \frac{1}{\sqrt{3}} \mathcal{P}_- (\Psi_D)_a^i \delta_i^a, \quad (4)$$

$$\tilde{\phi}^A = \frac{1}{\sqrt{2}} \mathcal{P}_+ (\Psi_D^C)_j^b (\lambda^A)_b^j, \quad \tilde{\phi} = \frac{1}{\sqrt{3}} \mathcal{P}_+ (\Psi_D^C)_j^b \delta_b^j, \quad (5)$$

where  $A = 1, \dots, 8$ , the sum over repeated indices is understood, and  $\mathcal{P}_\pm = (1 \pm \gamma^5)/2$  are the left- and right-handed projectors. A tilde denotes the charge conjugate spinors.

The order parameter in Eq. (1) is recovered by assuming that the following (singlet under the locked symmetry) vacuum expectation values are nonzero

$$\langle 0 | \bar{\psi} \tilde{\psi} | 0 \rangle = -\langle 0 | \bar{\phi} \tilde{\phi} | 0 \rangle = -\frac{1}{2} (3\kappa_1 + \kappa_2), \quad (6)$$

$$\langle 0 | \bar{\psi}^A \tilde{\psi}^B | 0 \rangle = -\langle 0 | \bar{\phi}^A \tilde{\phi}^B | 0 \rangle = -\frac{1}{2} \delta^{AB} \kappa_2. \quad (7)$$

A specific vacuum alignment leads to a relation between  $\kappa_1$  and  $\kappa_2$ . For example, if the condensate were pure antitriplet in color and antitriplet in flavor (with respect to the original symmetries of the action), it would imply that  $\kappa_2 = -\kappa_1$ . Similarly, in the case of a sextet in color and sextet in flavor condensate, the relation would read  $\kappa_2 = \kappa_1$ . It is known, however, that the true vacuum alignment is such that the antitriplet–antitriplet contribution dominates. At the same time, the sextet–sextet contribution is small but nonzero [12,15,16].

## III. SCHWINGER-DYSON EQUATION

In this section we briefly review the Schwinger-Dyson (SD) equation using our notation. This would also serve as a convenient reference point when we discuss the Bethe-Salpeter (BS) equations in Sec. V.

To start with, let us introduce the multicomponent spinor,

$$\begin{pmatrix} \Psi \\ \Psi^A \end{pmatrix}, \quad (8)$$

built of the left-handed Majorana spinors,

$$\Psi = \psi + \tilde{\psi}, \quad (9)$$

$$\Psi^A = \psi^A + \tilde{\psi}^A. \quad (10)$$

Similarly, we could introduce the multispinors made of the right-handed Majorana fields,  $\Phi$  and  $\Phi^A$ . In our analysis, restricted only to the (hard dense loop improved) rainbow approximation, the left and right sectors of the theory completely decouple. Then, without loss of generality, it is sufficient to study the SD equation only in one of the sectors.

The benefit of using the notation in Eq. (8) is that the inverse full propagator of quarks takes a very simple block-diagonal form,

$$G^{-1}(p) = \begin{pmatrix} S_1^{-1}(p) & 0 \\ 0 & \delta^{AB} S_2^{-1}(p) \end{pmatrix}, \quad (11)$$

$$S_1^{-1}(p) = -i(\not{p} + \mu \gamma^0 \gamma^5 + \Delta_1 \mathcal{P}_- + \bar{\Delta}_1 \mathcal{P}_+), \quad (12)$$

$$S_2^{-1}(p) = -i(\not{p} + \mu \gamma^0 \gamma^5 - \Delta_2 \mathcal{P}_- - \tilde{\Delta}_2 \mathcal{P}_+). \quad (13)$$

Here  $\Delta_{1,2} = \Delta_{1,2}^+ \Lambda_p^+ + \Delta_{1,2}^- \Lambda_p^-$  and  $\tilde{\Delta}_{1,2} = \gamma^0 \Delta_{1,2}^\dagger \gamma^0$ , and the “on-shell” projectors of quarks,

$$\Lambda_p^\pm = \frac{1}{2} \left( 1 \pm \frac{\vec{\alpha} \cdot \vec{p}}{|\vec{p}|} \right), \quad \vec{\alpha} = \gamma^0 \vec{\gamma}, \quad (14)$$

are the same as in Ref. [15]. We note that the gaps,  $\Delta_1$  and  $\Delta_2$ , enter the propagators (12) and (13) with opposite signs.

After expressing the standard bare vertex of QCD in terms of Majorana spinors (9) and (10), we arrive at the following matrix form of the vertex:

$$\gamma^{A\mu} = \gamma^\mu \begin{pmatrix} 0 & \frac{1}{\sqrt{6}} \delta^{AC} \gamma^5 \\ \frac{1}{\sqrt{6}} \delta^{AB} \gamma^5 & \frac{1}{2} (d^{ABC} \gamma^5 - i f^{ABC}) \end{pmatrix}, \quad (15)$$

where  $d^{ABC}$  and  $f^{ABC}$  are defined by the (anti-) commutation relations of the Gell-Mann matrices,

$$\{\lambda^A, \lambda^B\} = \frac{4}{3} \delta^{AB} + 2 d^{ABC} \lambda^C, \quad (16)$$

$$[\lambda^A, \lambda^B] = 2 i f^{ABC} \lambda^C. \quad (17)$$

With all the ingredients at hand, it is straightforward to derive the matrix form of the SD equation,

$$G^{-1}(p) = G_0^{-1}(p) + 4 \pi \alpha_s \int \frac{d^4 q}{(2\pi)^4} \gamma^{A\mu} G(q) \times \Gamma^{A\nu}(q, p) \mathcal{D}_{\mu\nu}(q - p). \quad (18)$$

Here  $\gamma^{A\mu}$  and  $\Gamma^{A\mu}$  are the bare and the full vertices, respectively. The gluon propagator in the SD equation is the same as in Ref. [15], except that the Meissner effect should, in general, be taken into account.

By inverting the expression in Eq. (11), we obtain the following representation for the quark propagator:

$$G(p) = \begin{pmatrix} S_1(p) & 0 \\ 0 & \delta^{AB} S_2(p) \end{pmatrix}, \quad (19)$$

where

$$\begin{aligned} S_1(p) = & i \frac{\gamma^0(p_0 + \epsilon_p^+) - \Delta_1^+}{p_0^2 - (\epsilon_p^+)^2 - |\Delta_1^+|^2} \Lambda_p^- \mathcal{P}_+ \\ & + i \frac{\gamma^0(p_0 - \epsilon_p^+) - (\Delta_1^+)^*}{p_0^2 - (\epsilon_p^+)^2 - |\Delta_1^+|^2} \Lambda_p^+ \mathcal{P}_- \\ & + i \frac{\gamma^0(p_0 - \epsilon_p^-) - \Delta_1^-}{p_0^2 - (\epsilon_p^-)^2 - |\Delta_1^-|^2} \Lambda_p^+ \mathcal{P}_+ \\ & + i \frac{\gamma^0(p_0 + \epsilon_p^-) - (\Delta_1^-)^*}{p_0^2 - (\epsilon_p^-)^2 - |\Delta_1^-|^2} \Lambda_p^- \mathcal{P}_-, \end{aligned} \quad (20)$$

$$\begin{aligned} S_2(p) = & i \frac{\gamma^0(p_0 + \epsilon_p^+) + \Delta_2^+}{p_0^2 - (\epsilon_p^+)^2 - |\Delta_2^+|^2} \Lambda_p^- \mathcal{P}_+ \\ & + i \frac{\gamma^0(p_0 - \epsilon_p^+) + (\Delta_2^+)^*}{p_0^2 - (\epsilon_p^+)^2 - |\Delta_2^+|^2} \Lambda_p^+ \mathcal{P}_- \\ & + i \frac{\gamma^0(p_0 - \epsilon_p^-) + \Delta_2^-}{p_0^2 - (\epsilon_p^-)^2 - |\Delta_2^-|^2} \Lambda_p^+ \mathcal{P}_+ \\ & + i \frac{\gamma^0(p_0 + \epsilon_p^-) + (\Delta_2^-)^*}{p_0^2 - (\epsilon_p^-)^2 - |\Delta_2^-|^2} \Lambda_p^- \mathcal{P}_-. \end{aligned} \quad (21)$$

The bare propagator in Eq. (18) is similar but with zero values of the gaps  $\Delta_n^\pm$  ( $n=1,2$ ).

In the (hard dense loop improved) rainbow approximation, both vertices in the SD equation are bare. By making use of the propagator in Eq. (19) and the vertex in Eq. (15), we derive the set of two gap equations [15,16]

$$\Delta_1(p) = -\frac{16}{3} \pi \alpha_s \int \frac{d^4 q}{(2\pi)^4} \gamma^\mu \mathcal{P}_+ S_2(q) \mathcal{P}_+ \gamma^\nu \mathcal{D}_{\mu\nu}(q - p), \quad (22)$$

$$\begin{aligned} \Delta_2(p) = & \frac{2}{3} \pi \alpha_s \int \frac{d^4 q}{(2\pi)^4} \gamma^\mu \mathcal{P}_+ [2 S_2(q) \\ & - S_1(q)] \mathcal{P}_+ \gamma^\nu \mathcal{D}_{\mu\nu}(q - p), \end{aligned} \quad (23)$$

where we used the identities

$$d^{ACD} d^{BCD} = \frac{5}{3} \delta^{AB}, \quad (24)$$

$$f^{ACD} f^{BCD} = 3 \delta^{AB}. \quad (25)$$

An approximate analytical solution to this coupled system of gap equations was presented in Ref. [15] (a numerical solution was also given in Ref. [16]). Here we quote the final results [15]:

$$\Delta_1^-(p_4) \approx 2 \Delta_2^-(p_4) \approx 2 \Delta_{(\bar{3}, \bar{3})}^-(p_4), \quad (26)$$

where

$$\Delta_{(\bar{3}, \bar{3})}^-(p_4) \approx \Delta_0, \quad p_4 \leq \Delta_0, \quad (27a)$$

$$\begin{aligned} \Delta_{(\bar{3}, \bar{3})}^-(p_4) \approx & \Delta_0 \sin \left( \sqrt{\frac{2\alpha_s}{9\pi}} \ln \frac{\Lambda}{p_4} \right), \\ & p_4 \geq \Delta_0, \end{aligned} \quad (27b)$$

with  $\Lambda \equiv 16(2\pi)^{3/2} \mu / (3\alpha_s)^{5/2}$  and

$$\Delta_0 \approx \frac{16(2\pi)^{3/2} \mu}{(3\alpha_s)^{5/2}} \exp \left( -\frac{3\pi^{3/2}}{2^{3/2} \sqrt{\alpha_s}} \right). \quad (28)$$

As in the case of two flavor dense QCD, the issue of the overall constant factor in this expression is still unsettled.

Some sources of possible corrections are discussed in Refs. [11,17]. In addition, we could argue (along the same lines as in Appendix B of Ref. [22]) that there is also at least one nonperturbative correction that could modify the constant factor in the expression for the gap.

#### IV. WARD IDENTITIES

In order to preserve the gauge invariance in dense QCD, one has to make sure that Green functions satisfy the Ward identities. In this section, we consider the simplest Ward identities that relate the vertex functions and the quark propagators. In addition to establishing the longitudinal part of the full vertex function, these identities will play a very important role in our analysis of the BS equations for the NG bosons.

In general, the structure of Ward identities in nonAbelian gauge theories (the Slavnov-Taylor identities) is much more complicated than in Abelian ones: they include contributions of the Faddeev-Popov ghosts. Fortunately in the (hard dense loop improved) ladder approximation, used in this paper, the situation simplifies. Indeed, since the direct interactions between gluons are neglected in this approximation, the Ward identities have an Abelian-type structure.

To start with, let us rewrite the conserved currents (related to the baryon number and color symmetry) in terms of the Weyl spinors, defined in Eqs. (9) and (10). In this approximation, the left- and right-handed sectors of the theory decouple and the axial  $U(1)_A$  charge is conserved. Therefore we can consider the (approximately) conserved currents in the two sectors separately. Here we give the details of the analysis for the left sector. The expressions for the singlet (related to the baryon number and the axial charge) and the octet (related to the color charge) channels read

$$j_\mu(x) = \bar{\Psi}_D(x) \gamma_\mu \mathcal{P}_+ \Psi_D(x) \\ = \frac{1}{2} \bar{\Psi}^A(x) \gamma_\mu \gamma^5 \Psi^A(x) + \frac{1}{2} \bar{\Psi}(x) \gamma_\mu \gamma^5 \Psi(x), \quad (29)$$

$$j_\mu^A(x) = \frac{1}{2} \bar{\Psi}_D(x) \gamma_\mu \lambda^A \mathcal{P}_+ \Psi_D(x) \\ = \frac{1}{4} \bar{\Psi}^B(x) \gamma_\mu (d^{ABC} \gamma^5 - i f^{ABC}) \Psi^C(x) \\ + \frac{1}{2\sqrt{6}} [\bar{\Psi}(x) \gamma_\mu \gamma^5 \Psi^A(x) + \bar{\Psi}^A(x) \gamma_\mu \gamma^5 \Psi(x)]. \quad (30)$$

Similar expressions could be written in the right-handed sector too. Now, we are interested in the Ward identities that relate the quark-gluon vertices to the propagators of quarks. Therefore, let us introduce the following (nonamputated) vertex functions:

$$\Gamma_\mu(x, y) = \langle 0 | T j_\mu(0) \Psi(x) \bar{\Psi}(y) | 0 \rangle, \quad (31a)$$

$$\Gamma_\mu^{AB}(x, y) = \langle 0 | T j_\mu(0) \Psi^A(x) \bar{\Psi}^B(y) | 0 \rangle, \quad (31b)$$

$$\Gamma_\mu^{A,BC}(x, y) = \langle 0 | T j_\mu^A(0) \Psi^B(x) \bar{\Psi}^C(y) | 0 \rangle, \quad (31c)$$

$$\Gamma_{1,\mu}^{A,B}(x, y) = \langle 0 | T j_\mu^A(0) \Psi^B(x) \bar{\Psi}(y) | 0 \rangle, \quad (31d)$$

$$\Gamma_{2,\mu}^{A,B}(x, y) = \langle 0 | T j_\mu^A(0) \Psi(x) \bar{\Psi}^B(y) | 0 \rangle. \quad (31e)$$

In order to derive the Ward identities, one needs to know the transformation properties of the quark fields. By making use of the transformation properties of the Dirac spinors, it is straightforward to derive the following infinitesimal baryon conservation symmetry transformations for the spinors of interest:

$$\delta \Psi^A = i \omega \gamma^5 \Psi^A, \quad (32)$$

$$\delta \bar{\Psi}^A = i \omega \bar{\Psi}^A \gamma^5, \quad (33)$$

$$\delta \Psi = i \omega \gamma^5 \Psi, \quad (34)$$

$$\delta \bar{\Psi} = i \omega \bar{\Psi} \gamma^5, \quad (35)$$

as well as the following color symmetry transformations:

$$\delta \Psi^A = \frac{i}{2} \omega^B (d^{ABC} \gamma^5 + i f^{ABC}) \Psi^C + \frac{i \omega^A}{\sqrt{6}} \gamma^5 \Psi, \quad (36)$$

$$\delta \bar{\Psi}^A = \frac{i}{2} \omega^B \bar{\Psi}^C (d^{ABC} \gamma^5 + i f^{ABC}) + \frac{i \omega^A}{\sqrt{6}} \bar{\Psi} \gamma^5, \quad (37)$$

$$\delta \Psi = \frac{i \omega^A}{\sqrt{6}} \gamma^5 \Psi^A, \quad (38)$$

$$\delta \bar{\Psi} = \frac{i \omega^A}{\sqrt{6}} \bar{\Psi}^A \gamma^5. \quad (39)$$

Now, by making use of the current conservation as well as the definition of the vertices in Eq. (31), we straightforwardly derive the Ward identities for the nonamputated vertices:

$$P^\mu \Gamma_\mu(k+P, k) = i [\gamma^5 S_1(k) + S_1(k+P) \gamma^5], \quad (40a)$$

$$P^\mu \Gamma_\mu^{AB}(k+P, k) = i \delta^{AB} [\gamma^5 S_2(k) + S_2(k+P) \gamma^5], \quad (40b)$$

$$P^\mu \Gamma_\mu^{A,BC}(k+P, k) = \frac{i}{2} d^{ABC} [\gamma^5 S_2(k) + S_2(k+P) \gamma^5] \\ - \frac{1}{2} f^{ABC} [S_2(k) - S_2(k+P)], \quad (40c)$$

$$P^\mu \Gamma_{1,\mu}^{A,B}(k+P,k) = \frac{i}{\sqrt{6}} \delta^{AB} [\gamma^5 S_1 k + S_2 (k+P) \gamma^5], \quad (40d)$$

$$P^\mu \Gamma_{2,\mu}^{A,B}(k+P,k) = \frac{i}{\sqrt{6}} \delta^{AB} [\gamma^5 S_2 k + S_1 (k+P) \gamma^5], \quad (40e)$$

where  $S_1$  and  $S_2$  are the quark propagators. In the leading order approximation where the wave function renormalization corrections are neglected, the explicit form of the momentum space propagators is given in Eqs. (20) and (21).

At this point, let us note that the use of the nonamputated vertices was crucial for the derivation of the Ward identities. Other than that, the nonamputated vertices are not very convenient to work with. In fact, it is the amputated rather than the nonamputated vertices that are usually used in the Feynman diagrams. Similarly, it is the amputated vertices that will appear in the BS equation in Sec. V. The formal definitions of the amputated vertices read

$$\Gamma_\mu(k+P,k) = S_1^{-1}(k+P) \Gamma_\mu(k+P,k) S_1^{-1}(k), \quad (41a)$$

$$\Gamma_\mu^{AB}(k+P,k) = S_2^{-1}(k+P) \Gamma_\mu^{AB}(k+P,k) S_2^{-1}(k), \quad (41b)$$

$$\Gamma_\mu^{A,BC}(k+P,k) = S_2^{-1}(k+P) \Gamma_\mu^{A,BC}(k+P,k) S_2^{-1}(k), \quad (41c)$$

$$\Gamma_{1,\mu}^{A,B}(k+P,k) = S_2^{-1}(k+P) \Gamma_{1,\mu}^{A,B}(k+P,k) S_1^{-1}(k), \quad (41d)$$

$$\Gamma_{2,\mu}^{A,B}(k+P,k) = S_1^{-1}(k+P) \Gamma_{2,\mu}^{A,B}(k+P,k) S_2^{-1}(k). \quad (41e)$$

It follows from Eq. (40) that they satisfy the following identities of their own:

$$P^\mu \Gamma_\mu(k+P,k) = i [S_1^{-1}(k+P) \gamma^5 + \gamma^5 S_1^{-1}(k)], \quad (42a)$$

$$P^\mu \Gamma_\mu^{AB}(k+P,k) = i \delta^{AB} [S_2^{-1}(k+P) \gamma^5 + \gamma^5 S_2^{-1}(k)], \quad (42b)$$

$$\begin{aligned} P^\mu \Gamma_\mu^{A,BC}(k+P,k) &= \frac{i}{2} d^{ABC} [S_2^{-1}(k+P) \gamma^5 + \gamma^5 S_2^{-1}(k)] \\ &\quad - \frac{1}{2} f^{ABC} [S_2^{-1}(k+P) - S_2^{-1}(k)], \end{aligned} \quad (42c)$$

$$\begin{aligned} P^\mu \Gamma_{1,\mu}^{A,B}(k+P,k) &= \frac{i}{\sqrt{6}} \delta^{AB} [S_2^{-1}(k+P) \gamma^5 \\ &\quad + \gamma^5 S_1^{-1}(k)], \end{aligned} \quad (42d)$$

$$P^\mu \Gamma_{2,\mu}^{A,B}(k+P,k) = \frac{i}{\sqrt{6}} \delta^{AB} [S_1^{-1}(k+P) \gamma^5 + \gamma^5 S_2^{-1}(k)]. \quad (42e)$$

In the rest of the paper, we are going to use these Ward identities a number of times. Because of a relatively simple structure of the inverse quark propagators, the last form of the identities for the amputated vertices will be particularly convenient.

Notice that in the limit  $P \rightarrow 0$ , we obtain

$$P^\mu \Gamma_\mu(k+P,k)|_{P \rightarrow 0} = 2(\tilde{\Delta}_1(k) \mathcal{P}_+ - \Delta_1(k) \mathcal{P}_-), \quad (43a)$$

$$P^\mu \Gamma_\mu^{AB}(k+P,k)|_{P \rightarrow 0} = 2 \delta^{AB} (\Delta_2(k) \mathcal{P}_- - \tilde{\Delta}_2(k) \mathcal{P}_+), \quad (43b)$$

$$P^\mu \Gamma_\mu^{A,BC}(k+P,k)|_{P \rightarrow 0} = d^{ABC} (\Delta_2(k) \mathcal{P}_- - \tilde{\Delta}_2(k) \mathcal{P}_+), \quad (43c)$$

$$\begin{aligned} P^\mu \Gamma_{1,\mu}^{A,B}(k+P,k)|_{P \rightarrow 0} &= \frac{1}{\sqrt{6}} \delta^{AB} [(\tilde{\Delta}_1(k) - \tilde{\Delta}_2(k)) \mathcal{P}_+ \\ &\quad + (\Delta_2(k) - \Delta_1(k)) \mathcal{P}_-], \end{aligned} \quad (43d)$$

$$\begin{aligned} P^\mu \Gamma_{2,\mu}^{A,B}(k+P,k)|_{P \rightarrow 0} &= \frac{1}{\sqrt{6}} \delta^{AB} [(\tilde{\Delta}_1(k) - \tilde{\Delta}_2(k)) \mathcal{P}_+ \\ &\quad + (\Delta_2(k) - \Delta_1(k)) \mathcal{P}_-]. \end{aligned} \quad (43e)$$

These expressions show that all five vertices have poles at  $P=0$ . Such poles indicate the presence of NG bosons that correspond to broken continuous symmetries in the theory.

Let us clarify the exact origin of the poles in all the vertices introduced. It is straightforward to show that the pole contributions in the first two vertices, defined in Eqs. (31a) and (31b), indicate the appearance of the NG boson related to breaking of the baryon number as well as the axial symmetry: in this approximation, the  $U(1)_A$  charge is conserved and the axial symmetry is spontaneously broken. This boson is a singlet with respect to the unbroken  $SU(3)_{L+R+c}$  subgroup. Similarly, one can see that the poles of the vertices in Eqs. (31c), (31d), and (31e) are due to an octet of the NG bosons related to breaking of the color symmetry (because of the locking, the chiral symmetry is also broken). Notice that considering the right-handed sector will double the number of the NG bosons.

One could also establish that the explicit pole structure of the vertices should read

$$\Gamma_\mu(k+P,k)|_{P \rightarrow 0} = \frac{P_\mu^{(\eta)} F^{(\eta)}}{P_\nu^{(\eta)} P^\nu} \boldsymbol{\eta}(k,0), \quad (44a)$$

$$\Gamma_\mu^{AB}(k+P,k)|_{P \rightarrow 0} = \frac{P_\mu^{(\eta)} F^{(\eta)}}{P_\nu^{(\eta)} P^\nu} \delta^{AB} \boldsymbol{\eta}'(k,0), \quad (44b)$$

$$\Gamma_\mu^{A,BC}(k+P,k)|_{P \rightarrow 0} = \frac{P_\mu^{(\pi)} F^{(\pi)}}{P_\nu^{(\pi)} P^\nu} d^{ABC} \boldsymbol{\pi}_0(k,0), \quad (44c)$$

$$\Gamma_{1,\mu}^{A,B}(k+P,k)|_{P \rightarrow 0} = \frac{P_\mu^{(\pi)} F^{(\pi)}}{P_\nu^{(\pi)} P^\nu} \delta^{AB} \pi_1(k,0), \quad (44d)$$

$$\Gamma_{2,\mu}^{A,B}(k+P,k)|_{P \rightarrow 0} = \frac{P_\mu^{(\pi)} F^{(\pi)}}{P_\nu^{(\pi)} P^\nu} \delta^{AB} \pi_2(k,0), \quad (44e)$$

where we introduced the notation  $P_\mu^{(x)} = (P_0, -c_x^2 \vec{P})$  (with  $c_x$  being the velocity of the appropriate NG boson). The decay constants are defined by

$$\langle 0 | j_\mu(0) | P \rangle = i P_\mu^{(\eta)} F^{(\eta)}, \quad (45a)$$

$$\langle 0 | j_\mu^A(0) | P; B \rangle = i \delta^{AB} P_\mu^{(\pi)} F^{(\pi)}. \quad (45b)$$

The BS wave functions in the coordinate representation read

$$\eta(x,y;P) \equiv e^{-iP(x+y)/2} \eta(x-y;P) = \langle 0 | T \Psi(x) \bar{\Psi}(y) | P \rangle, \quad (46a)$$

$$\begin{aligned} \eta'(x,y;P) &\equiv e^{-iP(x+y)/2} \eta'(x-y;P) \\ &= \frac{1}{8} \langle 0 | T \Psi^A(x) \bar{\Psi}^A(y) | P \rangle, \end{aligned} \quad (46b)$$

and

$$\begin{aligned} \pi_0^{A(B)}(x,y;P) &\equiv \delta^{AB} e^{-iP(x+y)/2} \pi_0(x-y;P) \\ &= \frac{3}{5} d^{ACD} \langle 0 | T \Psi^C(x) \bar{\Psi}^D(y) | P; B \rangle, \\ A, B &= 1, \dots, 8, \end{aligned} \quad (47a)$$

$$\begin{aligned} \pi_1^{A(B)}(x,y;P) &\equiv \delta^{AB} e^{-iP(x+y)/2} \pi_1(x-y;P) \\ &= \langle 0 | T \Psi^A(x) \bar{\Psi}(y) | P; B \rangle, \\ A, B &= 1, \dots, 8, \end{aligned} \quad (47b)$$

$$\begin{aligned} \pi_2^{A(B)}(x,y;P) &\equiv \delta^{AB} e^{-iP(x+y)/2} \pi_2(x-y;P) \\ &= \langle 0 | T \Psi(x) \bar{\Psi}^A(y) | P; B \rangle, \\ A, B &= 1, \dots, 8, \end{aligned} \quad (47c)$$

in the singlet and the octet channels, respectively. The momentum representation of the corresponding wave functions are given by the Fourier transforms of the translation invariant parts.

## V. BS EQUATIONS FOR NG BOSONS

The bound states and resonances should reveal themselves through the appearance of poles in Green functions. To consider the problem of diquark bound states in cold dense QCD, one has to introduce four-point Green functions that describe the two particle scattering in the diquark channels of interest. The residues at the poles of the Green functions are related to the BS wave functions of the bound states. By starting from the (inhomogeneous) BS equations

for the four-point Green functions, it is straightforward to derive the so-called homogeneous BS equations for the wave functions.

The NG bosons in the problem at hand are the composite diquark states. Altogether, there are 17 NG bosons and one pseudo-NG boson [the latter is related to breaking of the approximate  $U(1)_A$  symmetry]. As was already mentioned in Sec. IV, there is no difference between the pseudo NG boson and the real NG bosons in the ladder approximation. By combining the NG bosons in left and right sectors of the theory, we could construct the corresponding set of scalar and pseudoscalar states. We note, however, that the scalar octet is unphysical because of the Higgs mechanism. The pseudoscalar octet is built of real NG bosons related to the breaking of chiral symmetry. In addition, there is a scalar singlet and a pseudoscalar singlet. Both of them are physical, but the first is a real NG boson, while the other is a pseudo NG boson. While considering the BS equation in the left-handed sector, we could reveal only half of the total 18 NG bosons. As is clear, they should belong to singlet and octet representations of  $SU(3)_{L+R+c}$ .

We would like to note that though the scalars from the octet are removed from the physical spectrum by the Higgs mechanism, they exist in the theory as ‘‘ghosts’’ [29], and one cannot get rid of them completely, unless a unitary gauge is found. In fact these ghosts play an important role in getting rid of unphysical poles from on-shell scattering amplitudes [29]. We will use covariant gauges: because of the composite nature of the order parameter, it does not seem to be straightforward to define and to use the unitary gauge here.

The nine Majorana quark fields in Eq. (8) form a singlet and an octet with respect to the color-flavor locked group of the vacuum. Clearly, we could construct all kinds of different diquarks out of these building blocks. If there is an attraction in the appropriate channel, two constituent singlets could give rise to a diquark singlet. Similarly, one singlet and one octet could produce an octet of diquarks. Finally, out of two quark octets, one could construct a whole new set of additional diquarks. Indeed, by making use of the representation theory,

$$8 \otimes 8 = 1 \oplus 8_s \oplus 8_a \oplus 10 \oplus \overline{10} \oplus 27, \quad (48)$$

we might expect six more multiplets. As was shown in the preceding section, however, only the singlet and the first of the octets (the symmetric one) are relevant for the description of the NG bosons. One could also guess that the singlet and octet are the most attractive channels in the color-flavor locked phase of dense QCD. For this reason, we restrict our study of the general bound state problem to the analysis of only these two channels [though one should keep in mind a possibility of a mixing between those two octets (see Sec. V B)].

In order to derive the BS equations, we use the method developed in Ref. [22]. To this end, we need to use the following effective Lagrangian:

$$\begin{aligned}
\mathcal{L}_{\text{eff}} = & \frac{1}{2} \bar{\Psi} (\not{p} + \mu \gamma^0 \gamma^5 + \Delta_1 \mathcal{P}_- + \tilde{\Delta}_1 \mathcal{P}_+) \Psi \\
& + \frac{1}{2} \bar{\Psi}^A (\not{p} + \mu \gamma^0 \gamma^5 - \Delta_2 \mathcal{P}_- - \tilde{\Delta}_2 \mathcal{P}_+) \Psi^A \\
& + \frac{1}{4} \bar{\Psi}^B(x) A_\mu^A \gamma^\mu (d^{ABC} \gamma^5 - i f^{ABC}) \Psi^C(x) \\
& + \frac{1}{2\sqrt{6}} A_\mu^A (\bar{\Psi}(x) \gamma^\mu \gamma^5 \Psi^A(x) \\
& + \bar{\Psi}^A(x) \gamma^\mu \gamma^5 \Psi(x)), \tag{49}
\end{aligned}$$

plus the right-handed contribution. This effective Lagrangian is a starting point in the derivation of the BS equations for the wave functions introduced in Eqs. (46) and (47). While dealing with the multicomponent spinor defined in Eq. (8), it is rather natural to introduce the corresponding matrix form of the BS wave function. We use  $X(p; P)$  as a generic notation for such a matrix wave function.

In the (hard dense loop improved) ladder approximation, the matrix form of the BS equation reads

$$\begin{aligned}
G^{-1} \left( p + \frac{P}{2} \right) X(p; P) G^{-1} \left( p - \frac{P}{2} \right) \\
= -4\pi\alpha_s \int \frac{d^4 q}{(2\pi)^4} \gamma^{A\mu} X(q; P) \gamma^{B\nu} \mathcal{D}_{\mu\nu}^{AB}(q-p), \tag{50}
\end{aligned}$$

where  $\mathcal{D}_{\mu\nu}^{AB}(q-p)$  is the gluon propagator and  $\gamma^{A\mu}$  is the bare quark-gluon vertex given in Eq. (15). This approximation has the same status as the rainbow approximation in the SD equation. It assumes that the coupling constant is weak, and the leading perturbative expression for the kernel of the BS equation adequately represents the quark interactions.

#### A. BS equation in the singlet channel

Now, let us consider the BS equation in the singlet channel. As was explained above, it is sufficient to consider only the left-handed sector in this approximation. As we know from studying the pole structure of the vertices in Sec. IV, the singlet channel should contain at least one bound state, the diquark NG boson related to breaking of the baryon number [or, what is almost the same in our notation, to the approximate  $U(1)_A$  symmetry].

We start the derivation of the BS equation from establishing the structure of the wave function. By incorporating the definition in Eq. (46), we obtain the following matrix form of the (nonamputated) BS wave function in the singlet channel:

$$X_\eta(p, P) = \begin{pmatrix} \eta(p, P) & 0 \\ 0 & \delta^{AB} \eta'(p, P) \end{pmatrix}. \tag{51}$$

Now, we substitute this into the BS equation (50) and arrive at the following set of equations:

$$\begin{aligned}
S_1^{-1} \left( p + \frac{P}{2} \right) \eta(p, P) S_1^{-1} \left( p - \frac{P}{2} \right) \\
= \frac{16}{3} \pi\alpha_s \int \frac{d^4 q}{(2\pi)^4} \gamma^\mu \gamma^5 \eta'(q, P) \gamma^5 \gamma^\nu \mathcal{D}_{\mu\nu}(q-p), \tag{52}
\end{aligned}$$

$$\begin{aligned}
S_2^{-1} \left( p + \frac{P}{2} \right) \eta'(p, P) S_2^{-1} \left( p - \frac{P}{2} \right) \\
= \frac{2}{3} \pi\alpha_s \int \frac{d^4 q}{(2\pi)^4} \gamma^\mu \\
\times \left( \gamma^5 \eta(q, P) \gamma^5 + \frac{5}{2} \gamma^5 \eta'(q, P) \gamma^5 - \frac{9}{2} \eta' \right) \\
\times \gamma^\nu \mathcal{D}_{\mu\nu}(q-p). \tag{53}
\end{aligned}$$

In order to simplify the analysis, it is rather convenient to introduce the amputated BS wave functions,

$$\eta(p, P) = S_1^{-1} \left( p + \frac{P}{2} \right) \eta(p, P) S_1^{-1} \left( p - \frac{P}{2} \right), \tag{54a}$$

$$\eta'(p, P) = S_2^{-1} \left( p + \frac{P}{2} \right) \eta'(p, P) S_2^{-1} \left( p - \frac{P}{2} \right). \tag{54b}$$

In addition, let us restrict ourselves to the analysis of the BS wave functions in the limit  $P \rightarrow 0$ . As one could check, such a limit is well defined and it is consistent with the on-shell condition for the NG bosons. The BS equations, then, read

$$\begin{aligned}
\eta(p) = \frac{16}{3} \pi\alpha_s \int \frac{d^4 q}{(2\pi)^4} \gamma^\mu \gamma^5 S_2(q) \eta'(q) S_2(q) \gamma^5 \\
\times \gamma^\nu \mathcal{D}_{\mu\nu}(q-p), \tag{55}
\end{aligned}$$

$$\begin{aligned}
\eta'(p) = \frac{2}{3} \pi\alpha_s \int \frac{d^4 q}{(2\pi)^4} \gamma^\mu \left( \gamma^5 S_1(q) \eta(q) S_1(q) \gamma^5 \right. \\
+ \frac{5}{2} \gamma^5 S_2(q) \eta'(q) S_2(q) \gamma^5 \\
\left. - \frac{9}{2} S_2(q) \eta' S_2(q) \right) \gamma^\nu \mathcal{D}_{\mu\nu}(q-p). \tag{56}
\end{aligned}$$

Now, we would like to get a solution to these equations. As in the case of two flavor QCD [21,22], one could use the Ward identities to solve the problem.

By comparing the Ward identities in Eqs. (43a) and (43b) with the pole structure of the vertices in Eqs. (44a) and (44b), we derive the required structure of the BS wave functions in the singlet channel,

$$\begin{aligned}\eta(p) &= \frac{i}{F^{(\eta)}}(S_1^{-1}(p)\gamma^5 + \gamma^5 S_1^{-1}(p)) \\ &= \frac{2}{F^{(\eta)}}(\tilde{\Delta}_1(p)\mathcal{P}_+ - \Delta_1(p)\mathcal{P}_-),\end{aligned}\quad (57)$$

$$\begin{aligned}\eta'(p) &= \frac{i}{F^{(\eta)}}(S_2^{-1}(p)\gamma^5 + \gamma^5 S_2^{-1}(p)) \\ &= \frac{2}{F^{(\eta)}}(\Delta_2(p)\mathcal{P}_- - \tilde{\Delta}_2(p)\mathcal{P}_+).\end{aligned}\quad (58)$$

It is straightforward to show that this is indeed the solution to the set of the BS equations (55) and (56), provided that  $\Delta_1$  and  $\Delta_2$  are the solutions to the gap equations (22) and (23).

We use this solution in Sec. VI in order to derive the decay constants and velocities of the NG bosons.

### B. BS equation in the octet channel

By following the approach of the preceding subsection, let us also analyze the set of coupled BS equations in the octet channel. Again, it is clear from the pole structure of the vertices discussed in Sec. IV that this channel contains at least one solution which corresponds to the octet of diquark NG bosons.

We start from establishing the general structure of the matrix BS wave function. By making use of the definition in Eq. (47), we obtain the following form of the (nonamputated) wave function in the channel of interest:

$$X_{\pi}^C(p, P) = \begin{pmatrix} 0 & \delta^{BC}\pi_2(p, P) \\ \delta^{AC}\pi_1(p, P) & d^{ABC}\pi_0(p, P) + if^{ABC}\sigma(p, P) \end{pmatrix}. \quad (59)$$

Notice, that here we added the ‘‘antisymmetric’’ octet  $\sigma(p, P)$  [see Eq. (48)] to the general structure because this latter might in general mix with the NG octet. Of course, in the weakly coupled limit when the wave function renormalization is close to 1, the admixture of  $\sigma$  octet is expected to be negligible. The reason is that, according to the (approximate) Ward identity in Eq. (43c), the NG boson does not contain the antisymmetric contribution proportional to  $f^{ABC}$ . This is somewhat similar to the situation with the  $\sigma$  singlet in two flavor QCD [22].

After substituting the wave function (59) into the BS equation (50), we arrive at the following set of equations:

$$\begin{aligned}S_2^{-1}\left(p + \frac{P}{2}\right)\pi_1(p, P)S_1^{-1}\left(p - \frac{P}{2}\right) &= \frac{2}{3}\pi\alpha_s \int \frac{d^4q}{(2\pi)^4} \gamma^\mu \\ &\times \left( \gamma^5 \pi_2(q, P) \gamma^5 + \frac{5}{\sqrt{6}} \gamma^5 \pi_0(q, P) \gamma^5 + 3 \sqrt{\frac{3}{2}} \sigma(q, P) \gamma^5 \right) \gamma^\nu \mathcal{D}_{\mu\nu}(q - p),\end{aligned}\quad (60)$$

$$\begin{aligned}S_1^{-1}\left(p + \frac{P}{2}\right)\pi_2(p, P)S_2^{-1}\left(p - \frac{P}{2}\right) &= \frac{2}{3}\pi\alpha_s \int \frac{d^4q}{(2\pi)^4} \gamma^\mu \\ &\times \left( \gamma^5 \pi_1(q, P) \gamma^5 + \frac{5}{\sqrt{6}} \gamma^5 \pi_0(q, P) \gamma^5 - 3 \sqrt{\frac{3}{2}} \gamma^5 \sigma(q, P) \right) \gamma^\nu \mathcal{D}_{\mu\nu}(q - p),\end{aligned}\quad (61)$$

$$\begin{aligned}S_2^{-1}\left(p + \frac{P}{2}\right)\pi_0(p, P)S_2^{-1}\left(p - \frac{P}{2}\right) &= \frac{\pi\alpha_s}{2} \int \frac{d^4q}{(2\pi)^4} \gamma^\mu \\ &\times \left( 2 \sqrt{\frac{2}{3}} \gamma^5 [\pi_1(q, P) + \pi_2(q, P)] \gamma^5 - \gamma^5 \pi_0(q, P) \gamma^5 - 3 \pi_0(q, P) \right. \\ &\left. + 3 [\sigma(q, P) \gamma^5 - \gamma^5 \sigma(q, P)] \right) \gamma^\nu \mathcal{D}_{\mu\nu}(q - p),\end{aligned}\quad (62)$$

$$\begin{aligned}S_2^{-1}\left(p + \frac{P}{2}\right)\sigma(p, P)S_2^{-1}\left(p - \frac{P}{2}\right) &= \frac{\pi\alpha_s}{6} \int \frac{d^4q}{(2\pi)^4} \gamma^\mu (2\sqrt{6} [\pi_1(q, P) \gamma^5 - \gamma^5 \pi_2(q, P)] \\ &+ 5 \pi_0(q, P) \gamma^5 - 5 \gamma^5 \pi_0(q, P) + 5 \gamma^5 \sigma(q, P) \gamma^5 - 3 \sigma(q, P)) \gamma^\nu \mathcal{D}_{\mu\nu}(q - p).\end{aligned}\quad (63)$$

In deriving these equations, we used the following identities:

$$d^{AA'B'} d^{BB'C'} d^{CC'A'} = -\frac{1}{2} d^{ABC}, \quad (64)$$

$$d^{AA'B'} d^{BB'C'} f^{CC'A'} = -\frac{5}{6} f^{ABC}, \quad (65)$$

$$d^{AA'B'} f^{BB'C'} f^{CC'A'} = -\frac{3}{2} d^{ABC}, \quad (66)$$

$$f^{AA'B'} f^{BB'C'} f^{CC'A'} = \frac{3}{2} f^{ABC}. \quad (67)$$

Following the same approach as in the case of the singlet channel, we introduce the amputated wave functions by

$$\pi_1(p, P) = S_2^{-1} \left( p + \frac{P}{2} \right) \pi_1(p, P) S_1^{-1} \left( p - \frac{P}{2} \right), \quad (68a)$$

$$\pi_2(p, P) = S_1^{-1} \left( p + \frac{P}{2} \right) \pi_2(p, P) S_2^{-1} \left( p - \frac{P}{2} \right), \quad (68b)$$

$$\pi_0(p, P) = S_2^{-1} \left( p + \frac{P}{2} \right) \pi_0(p, P) S_2^{-1} \left( p - \frac{P}{2} \right), \quad (68c)$$

$$\sigma(p, P) = S_2^{-1} \left( p + \frac{P}{2} \right) \sigma(p, P) S_2^{-1} \left( p - \frac{P}{2} \right), \quad (68d)$$

and consider their BS equations in the limit of a vanishing total momentum of the diquark NG bosons,  $P \rightarrow 0$ . Therefore, we arrive at the following equations:

$$\begin{aligned} \pi_1(p) = & \frac{2}{3} \pi \alpha_s \int \frac{d^4 q}{(2\pi)^4} \gamma^\mu \\ & \times \left( \gamma^5 S_1(q) \pi_2(q) + \frac{5}{\sqrt{6}} \gamma^5 S_2(q) \pi_0(q) \right. \\ & \left. + 3 \sqrt{\frac{3}{2}} S_2(q) \sigma(q) \right) S_2(q) \gamma^5 \gamma^\nu \mathcal{D}_{\mu\nu}(q-p), \end{aligned} \quad (69)$$

$$\begin{aligned} \pi_2(p) = & \frac{2}{3} \pi \alpha_s \int \frac{d^4 q}{(2\pi)^4} \gamma^\mu \gamma^5 S_2(q) \\ & \times \left( \pi_1(q) S_1(q) \gamma^5 + \frac{5}{\sqrt{6}} \pi_0(q) S_2(q) \gamma^5 \right. \\ & \left. - 3 \sqrt{\frac{3}{2}} \sigma(q) S_2(q) \right) \gamma^\nu \mathcal{D}_{\mu\nu}(q-p), \end{aligned} \quad (70)$$

$$\begin{aligned} \pi_0(p) = & \frac{\pi \alpha_s}{2} \int \frac{d^4 q}{(2\pi)^4} \gamma^\mu \left( 2 \sqrt{\frac{2}{3}} \gamma^5 [S_2(q) \pi_1(q) S_1(q) \right. \\ & + S_1(q) \pi_2(q) S_2(q)] \gamma^5 - \gamma^5 S_2(q) \pi_0(q) S_2(q) \gamma^5 \\ & - 3 S_2(q) \pi_0(q) S_2(q) + 3 [S_2(q) \sigma(q) S_2(q) \gamma^5 \\ & \left. - \gamma^5 S_2(q) \sigma(q) S_2(q)] \right) \gamma^\nu \mathcal{D}_{\mu\nu}(q-p), \end{aligned} \quad (71)$$

$$\begin{aligned} \sigma(p) = & \frac{\pi \alpha_s}{6} \int \frac{d^4 q}{(2\pi)^4} \gamma^\mu (2 \sqrt{6} [S_2(q) \pi_1(q) S_1(q) \gamma^5 \\ & - \gamma^5 S_1(q) \pi_2(q) S_2(q)] + 5 S_2(q) \pi_0(q) S_2(q) \gamma^5 \\ & - 5 \gamma^5 S_2(q) \pi_0(q) S_2(q) + 5 \gamma^5 S_2(q) \sigma(q) S_2(q) \gamma^5 \\ & - 3 S_2(q) \sigma(q) S_2(q)) \gamma^\nu \mathcal{D}_{\mu\nu}(q-p). \end{aligned} \quad (72)$$

The approximate solution to this set of equations is obtained by comparing the Ward identities in Eqs. (43c), (43d), and (43e) with the pole structure of the vertices in Eqs. (44c), (44d), and (44e). In this way, we arrive at the following ansatz:

$$\begin{aligned} \pi_0(p) = & \frac{i}{2F(\pi)} (S_2^{-1}(p) \gamma^5 + \gamma^5 S_2^{-1}(p)) \\ = & \frac{1}{F(\pi)} (\Delta_2(p) \mathcal{P}_- - \tilde{\Delta}_2(p) \mathcal{P}_+), \end{aligned} \quad (73)$$

$$\begin{aligned} \pi_1(p) = & \frac{i}{\sqrt{6}F(\pi)} (S_2^{-1}(p) \gamma^5 + \gamma^5 S_1^{-1}(p)) \\ = & \frac{1}{\sqrt{6}F(\pi)} [(\tilde{\Delta}_1(p) - \tilde{\Delta}_2(p)) \mathcal{P}_+ \\ & + (\Delta_2(p) - \Delta_1(p)) \mathcal{P}_-], \end{aligned} \quad (74)$$

$$\begin{aligned} \pi_2(p) = & \frac{i}{\sqrt{6}F(\pi)} (S_1^{-1}(p) \gamma^5 + \gamma^5 S_2^{-1}(p)) \\ = & \frac{1}{\sqrt{6}F(\pi)} [(\tilde{\Delta}_1(p) - \tilde{\Delta}_2(p)) \mathcal{P}_+ \\ & + (\Delta_2(p) - \Delta_1(p)) \mathcal{P}_-], \end{aligned} \quad (75)$$

$$\sigma(p) = 0. \quad (76)$$

It is straightforward to show that this is a solution to the set of the BS equations (69)–(72). Notice, however, that the presented solution is approximate to the same extent as the quark propagators are. Indeed, if one takes into account the corrections due to the wave function renormalization of quarks, the expressions in Eqs. (74), (75) and (76), determined by the Ward identities, would be modified. It is remarkable that no similar modifications would appear for the singlet NG boson considered in the preceding subsection. This whole situation resembles the analysis in two flavor dense QCD [22], where the NG doublets were free of any admixtures, while the NG singlet had a contribution from another singlet. All the arguments of Ref. [22] in support of the self-consistency of the leading order approximation apply without changes to the analysis here.

## VI. DECAY CONSTANTS OF NG BOSONS

In this section, we derive the values of the decay constants for the NG bosons in the Pagels-Stokar approximation [30] (for a review see Ref. [31]). We start with the definitions in Eqs. (45a) and (45b). It is straightforward, then, to

derive the following exact expressions:

$$P_\mu^{(\eta)} F^{(\eta)} = \frac{i}{2} \int \frac{d^4 q}{(2\pi)^4} \text{tr} \{ \gamma_\mu \gamma^5 [8 \boldsymbol{\eta}'(q, P) + \boldsymbol{\eta}(q, P)] \}$$

$$= \frac{i}{2} \int \frac{d^4 q}{(2\pi)^4} \text{tr} \{ \gamma_\mu \gamma^5 [8 S_2(q + P/2) \boldsymbol{\eta}'(q, P) S_2(q - P/2) + S_1(q + P/2) \boldsymbol{\eta}(q, P) S_1(q - P/2)] \}, \quad (77)$$

$$P_\mu^{(\pi)} F^{(\pi)} = \frac{i}{12} \int \frac{d^4 q}{(2\pi)^4} \text{tr} \{ \gamma_\mu \gamma^5 (5 \boldsymbol{\pi}_0(q, P) + \sqrt{6} \boldsymbol{\pi}_1(q, P) + \sqrt{6} \boldsymbol{\pi}_2(q, P)) \}$$

$$= \frac{i}{12} \int \frac{d^4 q}{(2\pi)^4} \text{tr} \{ \gamma_\mu \gamma^5 (5 S_2(q + P/2) \boldsymbol{\pi}_0(q, P) S_2(q - P/2)$$

$$+ \sqrt{6} S_2(q + P/2) \boldsymbol{\pi}_1(q, P) S_1(q - P/2) + \sqrt{6} S_1(q + P/2) \boldsymbol{\pi}_2(q, P) S_2(q + P/2)) \}. \quad (78)$$

In order to calculate the integrals on the right-hand side, one needs to know the explicit form of the BS wave functions at nonzero values of the total momentum  $P$ . In general, however, it is hard to derive them. Therefore, it is natural to consider the Pagels-Stokar approximation [30,31]. Such an approximation uses the amputated wave functions at zero total momentum which are given in Eqs. (57) and (58) and Eqs. (73), (74), and (75) for the singlet and octet states, respectively. A simple calculation leads to the following result [see Eqs. (A1) and (A4) in the Appendix for details]:

$$(F^{(\eta)})^2 \left\{ \frac{P_0}{c_\eta^2 \vec{P}} \right\} \simeq \frac{9\mu^2}{2\pi^2} \left\{ \frac{P_0}{\frac{1}{3}\vec{P}} \right\}, \quad (79)$$

$$(F^{(\pi)})^2 \left\{ \frac{P_0}{c_\pi^2 \vec{P}} \right\} \simeq \frac{\mu^2}{24\pi^2} \left( 7 - \frac{|\Delta_1^-|^2 + |\Delta_2^-|^2 - |\Delta_1^- - \Delta_2^-|^2}{|\Delta_1^-|^2 - |\Delta_2^-|^2} \ln \frac{|\Delta_1^-|^2}{|\Delta_2^-|^2} \right) \left\{ \frac{P_0}{\frac{1}{3}\vec{P}} \right\} \simeq \frac{(21 - 8 \ln 2)\mu^2}{72\pi^2} \left\{ \frac{P_0}{\frac{1}{3}\vec{P}} \right\}, \quad (80)$$

where we used the relation between the gaps  $\Delta_1^- \approx 2\Delta_2^-$  as in Eq. (26).

Recall that here we consider the NG bosons from the left-handed sector. The expressions for the decay constants of the NG bosons from the right-handed sector are of course the same. In order to obtain the decay constants for the scalar and pseudoscalar NG bosons, one has to multiply the expressions for  $(F^{(\eta)})^2$  and  $(F^{(\pi)})^2$  in Eqs. (79) and (80) by a factor of 2.

We see that the decay constants of all NG bosons are of order  $\mu$ , and their velocities are equal to  $1/\sqrt{3}$ . After taking into account the difference in definitions of the decay constants, we find that expressions (79) and (80) agree with those derived in Ref. [24]<sup>1</sup> [as well as with those in Ref. [27], and up to a factor of 2 in  $(F^{(\pi)})^2$  with those in Ref. [28]], where a different approach was used.

A few words should be said about Ref. [25], also using the method of the BS equation for studying the NG bosons in

the CFL phase of cold dense QCD. The result for the decay constants obtained there is numerically different from our result as well as from the results of Refs. [24,27,28]. It seems the reason of that is because, while being qualitatively correct, the analysis of Ref. [25] misses some relevant quantitative details (for example, no distinction between the two types of quarks with different gaps seems to be made).

## VII. ABSENCE OF MASSIVE RADIAL EXCITATIONS

The important property of the quark pairing dynamics in cold dense QCD is the long range interaction mediated by the gluons of the magnetic type [5,6]. Of course, the Meissner effect would eventually provide screening for the gluons in the far infrared region. It is known that such screening is negligible (at least in the leading order) for the dynamics of the gap formation [7–11]. The pairing dynamics of the massive diquarks, however, is quite sensitive to the Meissner effect [21,22].

Let us now consider the difference between two flavor dense QCD and three flavor dense QCD. The former has the residual  $SU(2)_c$  gauge symmetry in the vacuum, so that three out of the total eight gluons do not feel the Meissner

<sup>1</sup>Our left-handed and right-handed NG fields correspond, respectively, to  $X$  and  $Y$  fields in the nonlinear realization of the  $SU(3)_c \times SU(3)_L \times SU(3)_R$  symmetry of Ref. [24].

effect. In contrast, the gauge symmetry of the latter is completely broken (through the Higgs mechanism). This means, in particular, that all eight gluons in the CFL phase of three flavor QCD are affected by the Meissner screening.

The natural question is whether the conjecture of Ref. [14] about the existence of an infinite tower of massive excitations in diquark channels with quantum numbers of the NG bosons could be generalized to three flavor QCD. We recall that this conjecture was derived from studying some unusual properties of the effective potential. Taken literally, the conjecture would imply the existence of an infinite tower of massive excitations for each of the five (would be) NG bosons in the case of QCD with two flavors. However, our detailed analysis [21,22] shows that, in fact, an infinite tower of the excitations occurs only in the color singlet channel. No radial excitations of the four other (would be) NG bosons (from a color doublet and antidoublet) appear. In order to reach this conclusion, one needs to take into account the effect of the Meissner screening for the pairing dynamics of diquark bound states.

By following the same arguments as in the case of two flavor QCD [21,22], one should distinguish between two classes of bound states, for which the role of the Meissner effect is very different. The first class consists of light bound states with the masses  $M \ll |\Delta_0^-|$ . The binding energy of these states is large (tightly bound states), and the Meissner effect is essentially irrelevant for their pairing dynamics. This point could be illustrated by the BS equations for the lightest diquarks, the massless NG bosons: in that case the most important region of momenta in the equations is given by  $|\Delta_0^-| \lesssim |k_0| \ll |\vec{k}| \lesssim \mu$  where the Meissner effect is negligible (see Sec. VII and Appendix C in Ref. [22]).

The second class includes quasiclassical states with the masses close to their threshold. Since the binding energy of the quasiclassical states is small, the quasiclassical part of the spectrum is almost completely determined by the behavior of the potential at large distances. In the particular case of cold dense QCD, the interaction between quarks is long ranged in the (imaginary) time direction and essentially short ranged in the spatial ones [6,14]. Because of that, the far infrared region, with  $|k_0| < |\Delta_0^-| \lesssim |\vec{k}|$ , is particularly important for the pairing dynamics of the quasiclassical diquark states and the Meissner effect is rather strong in that region. This implies that the inclusion of the Meissner effect is crucial for extracting the properties of the states from this second class (for more details see Sec. VII and Appendix C in Ref. [22]).

Now, by repeating the arguments of Ref. [22], we conclude that, because of the Meissner effect, an infinite tower of (quasiclassical) massive diquarks does not appear in the CFL phase of three flavor QCD. This is directly related to the fact that all eight gluons in the model at hand are affected by the Meissner screening. This qualitative argument by itself does not prevent the possibility of a finite number of radial excitations in the spectrum. However, the same type of analysis as in Ref. [22] shows that no radial excitations of NG bosons appear at all. After the Meissner effect is taken into account, the interaction provided by gluons appears to

be too weak to form even the lowest massive radial excitation of a NG boson.

## VIII. CONCLUSIONS

In this paper we studied the problem of diquarks with the quantum numbers of the NG bosons in the CFL phase of cold dense QCD with three quark flavors. For this purpose, we used the conventional method of the BS equation. Thus, we derived the general BS equations in the singlet and the octet channels that include all the NG bosons.

Our analytical analysis of the BS equations in CFL phase shows that the theory contains one pseudoscalar octet of the NG bosons, one singlet NG boson and one singlet pseudo NG boson. This agrees with the previous results obtained in the effective theory approach of Refs. [23–28]. We calculate the decay constants of the (pseudo) NG bosons by using the Pagels-Stokar approximation, and the results agree with those of Ref. [24], obtained by a very different method.

We also show that there are no spin-zero massive states (with quantum numbers of the NG bosons) in the CFL phase of cold dense QCD. This conclusion is reached by studying a specific realization of the Meissner effect in three flavor QCD. In contrast to the case of dense QCD with two flavors, all eight gluons in the CFL phase become massive due to the Meissner effect. As a result, the one-gluon interaction becomes rather weak in the far infrared region which is responsible for the pairing dynamics of the (would be quasiclassical) massive radial excitations of the NG bosons [22]. We note, though, that our current conclusions state nothing about the bound diquark states of higher spins. The problem of the higher spin channels might be also interesting, but it is beyond the scope of this paper.

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## APPENDIX: CALCULATION OF THE NG BOSON DECAY CONSTANTS

In this appendix we give the details of calculation of the decay constants. We start from the definitions in Eqs. (77) and (78). In the Pagels-Stokar approximation, the BS wave functions of the NG bosons are taken at vanishing total momentum  $P$ . The explicit form of the wave functions in the singlet channel is given in Eqs. (57) and (58). By making use of these expressions, we rewrite the right-hand side of Eq. (77) as follows:

$$\begin{aligned}
P_\mu^{(\eta)F^{(\eta)}} &\simeq \frac{i}{2F^{(\eta)}} \int \frac{d^4q}{(2\pi)^4} \text{tr}\{8\gamma_\mu S_2(q) \not{P} [S_2(q) \\
&\quad + \gamma^5 S_2(q) \gamma^5] + \gamma_\mu S_1(q) \not{P} [S_1(q) + \gamma^5 S_1(q) \gamma^5]\} + O(P^2) \\
&\simeq \frac{2}{F^{(\eta)}} P^\nu \int \frac{dq_4 d^3\vec{q}}{(2\pi)^4} \left( g_{\mu 0} g_{\nu 0} - \frac{\vec{q}_\mu \vec{q}_\nu}{|\vec{q}|^2} \right) \left( \frac{8|\Delta_2^-|^2}{[q_4^2 + (\epsilon_q^-)^2 + |\Delta_2^-|^2]^2} + \frac{|\Delta_1^-|^2}{[q_4^2 + (\epsilon_q^-)^2 + |\Delta_1^-|^2]^2} \right) + O(P^2) \\
&\simeq \frac{9\mu^2}{2\pi^2 F^{(\eta)}} \left( g_{\mu 0} P_0 + \frac{1}{3} \vec{P}_\mu \right) + O(P^2), \tag{A1}
\end{aligned}$$

where we used the expansion of the quark propagators in powers of momentum  $P$ ,

$$S_{1,2}(q + P/2) \simeq S_{1,2}(q) + \frac{i}{2} S_{1,2}(q) \not{P} S_{1,2}(q) + O(P^2), \tag{A2}$$

as well as the following results for the Dirac traces:

$$\text{tr}(\gamma_\mu \Lambda_q^\pm \gamma_\nu \Lambda_q^\mp \not{P}_\mp) = g_{\mu 0} g_{\nu 0} - \frac{\vec{q}_\mu \vec{q}_\nu}{|\vec{q}|^2}. \tag{A3}$$

In the case of the octet state, the calculation is similar. The explicit form of the corresponding BS wave functions is given in Eqs. (73), (74), and (75). By substituting them into Eq. (78), we arrive at

$$\begin{aligned}
P_\mu^{(\pi)F^{(\pi)}} &\simeq \frac{i}{24F^{(\pi)}} \int \frac{d^4q}{(2\pi)^4} \text{tr}\{5\gamma_\mu S_2(q) \not{P} [S_2(q) + \gamma^5 S_2(q) \gamma^5] \\
&\quad + 2\gamma_\mu [S_1(q) + \gamma^5 S_2(q) \gamma^5] \not{P} S_1(q) + 2\gamma_\mu [S_2(q) + \gamma^5 S_1(q) \gamma^5] \not{P} S_2(q)\} + O(P^2) \\
&\simeq \frac{1}{6F^{(\pi)}} P^\nu \int \frac{dq_4 d^3\vec{q}}{(2\pi)^4} \left( g_{\mu 0} g_{\nu 0} - \frac{\vec{q}_\mu \vec{q}_\nu}{|\vec{q}|^2} \right) \\
&\quad \times \left( \frac{5|\Delta_2^-|^2}{[q_4^2 + (\epsilon_q^-)^2 + |\Delta_2^-|^2]^2} + \frac{|\Delta_1^-|^2}{[q_4^2 + (\epsilon_q^-)^2 + |\Delta_1^-|^2]^2} + \frac{|\Delta_2^-|^2}{[q_4^2 + (\epsilon_q^-)^2 + |\Delta_2^-|^2]^2} \right. \\
&\quad \left. - \frac{\Delta_1^-(\Delta_2^-)^* + \Delta_2^-(\Delta_1^-)^*}{[q_4^2 + (\epsilon_q^-)^2 + |\Delta_1^-|^2][q_4^2 + (\epsilon_q^-)^2 + |\Delta_2^-|^2]} \right) + O(P^2) \\
&\simeq \frac{\mu^2}{24\pi^2 F^{(\pi)}} \left( 7 - \frac{\Delta_1^-(\Delta_2^-)^* + \Delta_2^-(\Delta_1^-)^*}{|\Delta_1^-|^2 - |\Delta_2^-|^2} \ln \frac{|\Delta_1^-|^2}{|\Delta_2^-|^2} \right) \left( g_{\mu 0} P_0 + \frac{1}{3} \vec{P}_\mu \right) + O(P^2). \tag{A4}
\end{aligned}$$

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